УДК 512.745, 512.815.4, 519.114

Bedratyuk L., Bedratyuk A.

[1] **Doctor of Sciences (Physics and Mathematics), Professor of Software Engineering Department, Khmelnytsky National University**

[2] **Senior Lecturer of the Department of Software Engineering, Khmelnytsky National University**

MAPLE PACKAGE FOR CALCULATING POINCARÉ SERIES.

We offer a Maple package `Poincare_Series` for calculating the Poincaré series for the algebras of invariants/covariants of binary forms, for the algebras of joint invariants/covariants of several binary forms, for the kernel of Weitzenböck derivations, for the bivariate Poincaré series of algebra of covariants of binary d-form and for the multivariate Poincaré series of the algebras of joint invariants/covariants of several binary forms.

Key words: *invariant, covariant, derivations, binary form, Poincaré series, algebra of invariants*

Introduction

Let V_d be the complex vector space of binary forms of degree d endowed with the natural action of the special linear group $G = SL_2$. Consider the corresponding action of the group G on the coordinate rings $\mathbb{C}[V_{\boldsymbol{d}}]$ and $\mathbb{C}[V_{\boldsymbol{d}} \oplus \mathbb{C}^2]$, where $V_{\boldsymbol{d}} := V_{d_1} \oplus V_{d_1} \oplus \cdots \oplus V_{d_n}$. Denote by $\mathcal{I}_{\boldsymbol{d}} = \mathbb{C}[V_{\boldsymbol{d}}]^G$ and by $\mathcal{C}_{\boldsymbol{d}} = \mathbb{C}[V_{\boldsymbol{d}} \oplus \mathbb{C}^2]^G$ the subalgebras of G-invariant polynomial functions. In the language of classical invariant theory the algebras $\mathcal{I}_{\boldsymbol{d}}$ and $\mathcal{C}_{\boldsymbol{d}}$ are called the algebra of join invariants and the algebra of join covariants for the n binary form of degrees $d_1, d_2, \ldots, d_n$ respectively. The coordinate ring $\mathbb{C}[V_{\boldsymbol{d}} \oplus \mathbb{C}^2]$ can be identified with the algebra of polynomials of the coefficients of the binary forms and of two subsidiary variables, say X, Y. The degree of a covariant with respect to the variables X, Y is called *the order* of the covariant. Every invariant has the order zero. The algebras $\mathcal{C}_{\boldsymbol{d}}$, $\mathcal{I}_{\boldsymbol{d}}$ are a finitely generated multigraded algebras under the multidegree-order:

$$\mathcal{C}_{\boldsymbol{d}} = (\mathcal{C}_{\boldsymbol{d}})_{\boldsymbol{m},0} + (\mathcal{C}_{\boldsymbol{d}})_{\boldsymbol{m},1} + \cdots + (\mathcal{C}_{\boldsymbol{d}})_{\boldsymbol{m},j} + \cdots ,$$

where each subspace $(\mathcal{C}_{\boldsymbol{d}})_{\boldsymbol{d},j}$ of covariants of multidegree $\boldsymbol{m} := (m_1, m_2, \ldots, m_n)$ and order j is finite-dimensional. The formal power series

$$\mathcal{P}(\mathcal{C}_{\boldsymbol{d}}, z_1, z_2, \ldots, z_n, t) = \sum_{\boldsymbol{m},j=0}^{\infty} \dim((\mathcal{C}_{\boldsymbol{d}})_{\boldsymbol{m},j}) z_1^{m_1} z_2^{m_2} \cdots z_n^{m_n} t^j,$$

$$\mathcal{P}(\mathcal{I}_{\boldsymbol{d}}, z_1, z_2, \ldots, z_n) = \sum_{\boldsymbol{m}}^{\infty} \dim((\mathcal{I}_{\boldsymbol{d}})_{\boldsymbol{m},j}) z_1^{m_1} z_2^{m_2} \cdots z_n^{m_n},$$

are called the multivariariate Poincaré series of the algebra of join covariants $\mathcal{C}_{\boldsymbol{d}}$. It is clear that the series $\mathcal{P}(\mathcal{C}_{\boldsymbol{d}}, z, z, \ldots, z, 0)$ is the Poincare series of the algebra $\mathcal{I}_{\boldsymbol{d}}$ and the series $\mathcal{P}(\mathcal{C}_{\boldsymbol{d}}, z, z, \ldots, z, 1)$ is the Poincare series of the algebra $\mathcal{C}_{\boldsymbol{d}}$ with respect to the usual grading of the algebra under degree. The algebra of covariants $\mathcal{C}_d$ is Cohen-Macaulay. It implies that its multivariate Poincaré series is the power series expansion of a rational function of the variables $z_1, \ldots, z_n, t$. We consider here the problem of computing efficiently this rational function.

In the paper we offer a Maple package for calculating the Poincaré series for such algebras of invariants:the Poincaré series for the algebras of invariants/covariants of binary forms, for the algebras of joint invariants/covariants of several binary forms, for the kernel of Weitzenböck derivations,for the bivariate Poincaré series of algebra of covariants of binary d-form and for the multivariate Poincaré series of the algebras of joint invariants/covariants of several binary forms. The present package implements results of the papers [1]–[6].

The package can be downloaded from the site
http://sites.google.com/site/bedratyuklp/.
To start, one proceeds as follows:

1. download the file `Poincare_Series.mpl` and save it into your Maple directory;
2. download the Xin's file (see a link at the web page) `Ell2.mpl` and save it into your Maple directory;
3. run Maple;
4. > `read "Poincare_Series.mpl": read "Ell2.mpl":`
5. If necessary then use > `Help();`

Formulas for the Poincaré series.

Below is the list of main formulas which are being implemented in the package.

Invariants and covariants of binary form

Let $\mathcal{I}_d$, $\mathcal{C}_d$ be algebras of invariants and covariants of binary d-form graded under degree. We have

$$\mathcal{P}(\mathcal{I}_d, z) = \sum_{0 \leqslant k < d/2} \varphi_{d-2\,k} \left(\frac{(-1)^k z^{k(k+1)}(1-z^2)}{(z^2,z^2)_k\,(z^2,z^2)_{d-k}} \right), \text{ (Springer's formula, see [9])}, \tag{1}$$

$$\mathcal{P}(\mathcal{C}_d, z) = \sum_{0 \leqslant k < d/2} \varphi_{d-2\,k} \left(\frac{(-1)^k z^{k(k+1)}(1+z)}{(z^2,z^2)_k\,(z^2,z^2)_{d-k}} \right), \tag{2}$$

here $(a,q)_n = (1-a)(1-a\,q)\cdots(1-a\,q^{n-1})$ denotes the q-shifted factorial and the function $\varphi_n : \mathbb{C}[[z]] \to \mathbb{C}[[z]]$ defined by

$$\varphi_n\left(\sum_{i=0}^{\infty} a_i z^i\right) = \sum_{i=0}^{\infty} a_{in} z^i.$$

Joint invariants and covariants of binary form

Let $\mathcal{I}_{\boldsymbol{d}}$, $\mathcal{C}_{\boldsymbol{d}}$, $\boldsymbol{d} = (d_1, d_2, \ldots, d_n)$ be algebras of joint invariants and joint covariants of n binary forms of degrees d_1, d_2, $\ldots, d_n$. Then

$$\mathcal{P}(\mathcal{I}_{\boldsymbol{d}}, z) = \sum_{i=0}^{d^*} \sum_{k=1}^{\beta_i} \frac{1}{(k-1)!} \frac{d^{k-1}\left(z^{k-1}\varphi_{d^*-k}((1-z^2)\,A_{i,k}(z))\right)}{dz^{k-1}}, \tag{3}$$

$$\mathcal{P}(\mathcal{C}_{\boldsymbol{d}}, z) = \sum_{i=0}^{d^*} \sum_{k=1}^{\beta_i} \frac{1}{(k-1)!} \frac{d^{k-1}\left(z^{k-1}\varphi_{d^*-k}((1+z)\,A_{i,k}(z))\right)}{dz^{k-1}}, \tag{4}$$

where

$$A_{i,k}(z) = \frac{(-1)^{\beta_i - k}}{(\beta_i - k)!\,(z^i)^{\beta_i - k}} \lim_{t \to z^{-i}} \frac{\partial^{\beta_i - k}}{\partial t^{\beta_i - k}} \left(f_d(tz^{d^*}, z)(1 - tz^i)^{\beta_i} \right).$$

The integer numbers $\beta_i, i = 0, \ldots, 2d^*$, $d^* := \max(d_1, d_2, \ldots, d_n)$, are defined from the decomposition

$$f_{\boldsymbol{d}}(tz^{d^*}, z) = \left((1-t)^{\beta_0}(1-tz)^{\beta_1}(1-tz^2)^{\beta_2} \ldots (1-tz^{2\,d^*})^{\beta_{2\,d^*}} \right)^{-1},$$

where

$$f_{\boldsymbol{d}}(t,z) = \left(\prod_{k=1}^{s} (tz^{-\,d_k}, z^2)_{d_k+1} \right)^{-1}.$$

Joint invariants and covariants of linear and quadratic binary forms

Let $d_1 = d_2 = \ldots = d_n = 1$, i.e. $\boldsymbol{d} = (1, 1, \ldots, 1)$. Then

$$\mathcal{P}(\mathcal{I}_{\boldsymbol{d}}, z) = \sum_{k=1}^{n} \frac{(-1)^{n-k}}{(k-1)!} \frac{(n)_{n-k}}{(n-k)!} \frac{d^{k-1}}{dz^{k-1}} \left(\left(\frac{z}{1-z^2} \right)^{2n-k-1} \right) = \tag{5}$$

$$= \frac{\mathrm{N}_{n-2}(z^2)}{(1-z^2)^{2n-3}} = \sum_{i=0}^{\infty} h_n(i) z^i, \tag{6}$$

where $\mathrm{N}_n(z)$ is the n-th Narayana polynomial of the first type

$$\mathrm{N}_n(z) = \sum_{k=1}^{n} \frac{1}{k} \binom{n-1}{k-1} \binom{n}{k-1} z^{k-1},$$

$h_n(i)$ is the Hilbert polynomial of the graded algebra $\mathcal{I}_{\boldsymbol{d}}$

$$h_n(i) = \frac{\cos\left(\frac{\pi\,i}{2}\right)^2}{4^{n-2}(n-2)!(n-1)!} (i+2)\,(i+2(n-1)) \left(\prod_{p=2}^{n-2} (i+2p) \right)^2,$$

and $(n)_m := n(n+1)\cdots(n+m-1)$, $(n)_0 := 1$ denotes the shifted factorial.

$$\mathcal{P}(\mathcal{C}_{\boldsymbol{d}}, z) = \sum_{k=1}^{n} \frac{(-1)^{n-k}}{(k-1)!} \frac{(n)_{n-k}}{(n-k)!} \frac{d^{k-1}}{dz^{k-1}} \left(\frac{(1+z)z^{2n-k-1}}{(1-z^2)^{2n-k}} \right) = \tag{7}$$

$$= \frac{W_{n-1}(z^2) + nzN_{n-1}(z^2)}{(1-z^2)^{2n-1}}, \tag{8}$$

where $\mathrm{W}_n(z)$ is the n-th Narayana polynomial of the second type.

Let $d_1 = d_2 = \ldots = d_n = 2$, $\boldsymbol{d} = (2, 2, \ldots, 2)$, then

$$\mathcal{P}(\mathcal{I}_{\boldsymbol{d}}, z) = \sum_{k=1}^{n} \frac{(-1)^{n-k}}{(n-k)(k-1)!} \frac{d^{k-1}}{dz^{k-1}} \left(\sum_{i=0}^{n-k} \binom{n-k}{i} \frac{(n)_i (n)_{n-k-i} (1-z) z^{2n-k-i-1}}{(1-z)^{n+i}(1-z^2)^{2n-k-i}} \right), \tag{9}$$

$$\mathcal{P}(\mathcal{C}_{\boldsymbol{d}}, z) = \sum_{k=1}^{n} \frac{(-1)^{n-k}}{(n-k)!(k-1)!} \frac{d^{k-1}}{dz^{k-1}} \left(\sum_{i=0}^{n-k} \binom{n-k}{i} \frac{(n)_i (n)_{n-k-i} z^{2n-k-i-1}}{(1-z)^{n+i}(1-z^2)^{2n-k-i}} \right) = \tag{10}$$

$$= \frac{\sum_{i=0}^{n-1} \binom{n-1}{i}^2 (z^2)^i}{(1-z)^n (1-z^2)^{2n-1}}. \tag{11}$$

Kernel of Weitzenbök derivation

Denote by $\mathcal{D}_{\boldsymbol{d}}$ the Weitzenbök derivation (linear locally nilpotent derivation) with its matrix consisting of n Jordan blocks of size d_1+1, d_2+1, $\ldots, d_s+1$, respectively. Since $\ker \mathcal{D}_{\boldsymbol{d}} \cong \mathcal{C}_{\boldsymbol{d}}$ and the isomorphism preserve degrees then have that $\mathcal{P}(\ker \mathcal{D}_{\boldsymbol{d}}, z) = \mathcal{P}(\mathcal{C}_{\boldsymbol{d}}, z)$.

Bivariate Poincare series for covariants of binary form

The algebra $\mathcal{C}_d$ of covariants is a finitely generated bigraded algebra:

$$\mathcal{C}_d = (\mathcal{C}_d)_{0,0} + (\mathcal{C}_d)_{1,0} + \cdots + (\mathcal{C}_d)_{i,j} + \cdots,$$

where each subspace $(\mathcal{C}_d)_{i,j}$ of covariants of degree i and order j is finite-dimensional. We have

$$\mathcal{P}(\mathcal{C}_d, z, t) = \sum_{i=0}^{\infty} (\mathcal{C}_d)_{i,j} z^i t^j = \sum_{0 \leqslant k < d/2} \psi_{d-2\,k} \left(\frac{(-1)^k t^{k(k+1)}(1-t^2)}{(t^2,t^2)_k \, (t^2,t^2)_{d-k}} \right) \frac{1}{1 - z t^{d-2\,k}}, \tag{12}$$

where $\psi_n : \mathbb{Z}[[t]] \to \mathbb{Z}[[t,z]], n \in \mathbb{Z}_+$ be a $\mathbb{C}$-linear function defined by

$$\psi_n\,(t^m) := \begin{cases} z^i t^j, \text{ if } m = n\,i - j, j < n, \\ 0, \text{ otherwise.} \end{cases}$$

Note that $\mathcal{P}(\mathcal{C}_d, z, 0) = \mathcal{P}(\mathcal{I}_d, z)$ and $\mathcal{P}(\mathcal{C}_d, z, 1) = \mathcal{P}(\mathcal{C}_d, z)$.

Multivariate Poincaré series

The algebra $\mathcal{C}_{\boldsymbol{d}}$ is a finitely generated multigraded algebra under the multidegree-order:

$$\mathcal{C}_{\boldsymbol{d}} = (\mathcal{C}_{\boldsymbol{d}})_{\boldsymbol{m},0} + (\mathcal{C}_{\boldsymbol{d}})_{\boldsymbol{m},1} + \cdots + (\mathcal{C}_{\boldsymbol{d}})_{\boldsymbol{m},j} + \cdots,$$

where each subspace $(\mathcal{C}_{\boldsymbol{d}})_{\boldsymbol{d},j}$ of covariants of multidegree $\boldsymbol{m} := (m_1, m_2, \ldots, m_n)$ and order j is finite-dimensional. The formal power series

$$\mathcal{P}(\mathcal{C}_{\boldsymbol{d}}, z_1, z_2, \ldots, z_n, t) = \sum_{\boldsymbol{m},j=0}^{\infty} \dim((\mathcal{C}_{\boldsymbol{d}})_{\boldsymbol{m},j}) z_1^{m_1} z_2^{m_2} \cdots z_n^{m_n} t^j,$$

is called the multivariariate Poincaré series of the algebra of join covariants $\mathcal{C}_{\boldsymbol{d}}$.

The following formula holds:

$$\mathcal{P}(\mathcal{C}_{\boldsymbol{d}}, z_1, z_2, \ldots, z_n, t) = \underset{\geqslant 0}{\Omega} f_{\boldsymbol{d}} \left(z_1 (t\lambda)^{d_1}, z_2 (t\lambda)^{d_2}, \ldots, z_s (t\lambda)^{d_s}, \frac{1}{t\lambda} \right),$$

where

$$f_{\boldsymbol{d}}(z_1, z_2, \ldots, z_n, t) = \frac{1}{\prod_{k=1}^{n} \prod_{j=0}^{d_k} (1 - z_k t^{d_k - 2j})},$$

For the multivariariate Poincaré series of the algebra of join invariants $\mathcal{I}_{\boldsymbol{d}}$ we have

$$\mathcal{P}(\mathcal{I}_{\boldsymbol{d}}, z_1, z_2, \ldots, z_n) = \underset{=0}{\Omega} f_{\boldsymbol{d}} \left(z_1 (t\lambda)^{d_1}, z_2 (t\lambda)^{d_2}, \ldots, z_s (t\lambda)^{d_s}, \frac{1}{t\lambda} \right).$$

Here $\underset{\geqslant 0}{\Omega}$ and $\underset{=0}{\Omega}$ are the MacMahon's Omega operators which act on Laurent series

$$\sum_{k_1=-\infty}^{\infty} \cdots \sum_{k_s=-\infty}^{\infty} \sum_{\alpha=-\infty}^{\infty} a_{k_1, k_2, \ldots, k_s, \alpha} z_1^{k_1} z_2^{k_2} \cdots z_s^{k_s} \lambda^{\alpha}$$

by

$$\underset{\geqslant 0}{\Omega} \sum_{k_1=-\infty}^{\infty} \cdots \sum_{k_s=-\infty}^{\infty} \sum_{\alpha=-\infty}^{\infty} a_{k_1, \ldots, k_s, \alpha} z_1^{k_1} \cdots z_s^{k_s} \lambda^{\alpha} = \sum_{k_1=0}^{\infty} \cdots \sum_{k_s=0}^{\infty} \sum_{\alpha=0}^{\infty} a_{k_1, \ldots, k_s, \alpha} z_1^{k_1} \cdots z_s^{k_s} \lambda^{\alpha},$$

and

$$\underset{=0}{\Omega} \sum_{k_1=-\infty}^{\infty} \cdots \sum_{k_s=-\infty}^{\infty} \sum_{\alpha=-\infty}^{\infty} a_{k_1, \ldots, k_s, \alpha} z_1^{k_1} \cdots z_s^{k_s} \lambda^{\alpha} = \sum_{k_1=0}^{\infty} \cdots \sum_{k_s=0}^{\infty} a_{k_1, \ldots, k_s, \alpha} z_1^{k_1} \cdots z_s^{k_s}.$$

Package Commands and Syntax

Command name: `INVARIANTS_SERIES`
Feature: Computes the Poincare series for the algebras of joint invariants for the binary forms of degrees $d_1, d_2, \ldots, d_n$.
Calling sequence: `INVARIANTS_SERIES` $([d_1, d_2, \ldots, d_n])$;
Parameters:
$[d_1, d_2, \ldots, d_n]$ - a list of degrees of n binary forms.
n - an integer, $n \geqslant 1$.

Command name: `COVARIANTS_SERIES`
Feature: Computes the Poincare series for the algebras of joint covariants for the binary forms of degrees $d_1, d_2, \ldots, d_n$.
Calling sequence: `COVARIANTS_SERIES` $([d_1, d_2, \ldots, d_n])$;
Parameters:
$[d_1, d_2, \ldots, d_n]$ - a list of degrees of n binary forms.
n - an integer, $n \geqslant 1$.

Command name: `KERNEL_SERIES`
Feature: Computes the Poincare series for the kernel of Weitzenböck derivation defined by n Jordan block of sizes $d_1 + 1$, $d_2 + 1$, $\ldots, d_n$.
Calling sequence: `KERNEL_SERIES` $([d_1, d_2, \ldots, d_n])$;
Parameters:
$[d_1, d_2, \ldots, d_n]$ - a list of sizes of the n Jordan blocks.
n - an integer, $n \geqslant 1$.

Command name: `BIVARIATE_SERIES`
Feature: Computes the bivariate Poincare series for the algebra of covariants of binary form of degree d. Also, computes the bivariate Poincare series for the kernel of the basic Weitzenböck derivation.
Calling sequence: `BIVARIATE_SERIES` $([d])$;
Parameters:
d - the degree of binary form.

Command name: `MULTIVAR_COVARIANTS`
Feature: Computes the multivariate Poincaré series for the algebra of joint covariants for n binary forms of degrees $d_1, d_2, \ldots, d_n$.

Calling sequence: `MULTIVAR_COVARIANTS` $([d_1, d_2, \ldots, d_n])$;
Parameters:
$[d_1, d_2, \ldots, d_n]$ - a list of degrees of n binary forms.
n - an integer, $n \geqslant 1$.

Command name: `MULTIVAR_INVARIANTS`
Feature: Computes the multivariate Poincaré series for the algebra of joint invariants for n binary forms of degrees $d_1, d_2, \ldots, d_n$.
Calling sequence: `MULTIVAR_INVARIANTS` $([d_1, d_2, \ldots, d_n])$;
Parameters:
$[d_1, d_2, \ldots, d_n]$ - a list of degrees of n binary forms.
n - an integer, $n \geqslant 1$.

Examples

Compute $\mathcal{P}(\mathcal{I}_6, z)$
Use the command

```
> INVARIANTS_SERIES([6]);
```

$$\frac{z^8 + z^7 - z^5 - z^4 - z^3 + z + 1}{(z^6 + z^5 + z^4 - z^2 - z - 1)(z^6 + z^5 - z - 1)(-1 + z^2)(-1 + z)}$$

Compute $\mathcal{P}(\mathcal{C}_6, z)$
Use the command

```
> COVARIANTS_SERIES([6]);
```

$$\frac{z^{10} + z^8 + 3z^7 + 4z^6 + 4z^5 + 4z^4 + 3z^3 + z^2 + 1}{(z^6 + z^5 + z^4 - z^2 - z - 1)(z^6 + z^5 - z - 1)(-1 + z^2)(-1 + z)^3}$$

Compute $\mathcal{P}(\mathcal{I}_{(1,2,3)}, z)$
Use the command

```
> INVARIANTS_SERIES([1,2,3]);
```

$$\frac{z^{12} + z^9 + 2z^8 + 3z^7 + 3z^6 + 3z^5 + 2z^4 + z^3 + 1}{(-1 + z^4)^2 (-1 + z^3)^2 (-1 + z)(-1 + z^2)(z^4 + z^3 + z^2 + z + 1)}$$

Compute $\mathcal{P}(\mathcal{C}_{(2,2,2)}, z)$
Use the command

```
> COVARIANTS_SERIES([2,2,2]);
```

$$\frac{z^4 + 4z^2 + 1}{(-1 + z)^3 (-1 + z^2)^5}$$

Compute $\mathcal{P}(\ker \mathcal{D}_{(4)}, z)$
Use the command

```
> KERNEL_SERIES([4]);
```

$$\frac{z^2 - z + 1}{(-1 + z^2)(-1 + z^3)(-1 + z)^2}$$

Compute $\mathcal{P}(\ker \mathcal{D}_{(1,1,1,2)}, z)$
Use the command

```
> KERNEL_SERIES([1,1,1,2]);
```

$$\frac{z^8 + 2z^7 + 7z^6 + 11z^5 + 11z^4 + 11z^3 + 7z^2 + 2z + 1}{(-1 + z^2)^3 (-1 + z^3)^3 (-1 + z)^2}$$

Compute $\mathcal{P}(\mathcal{C}_4, z, t)$

Use the command

```
> BIVARIATE_SERIES([4]);
```

$$\frac{t^4z^2 - zt^2 + 1}{(-1+zt^2)(-1+zt^4)(-1+z^2)(-1+z^3)}$$

Compute $\mathcal{P}(\mathcal{C}_{(1,1,2)}, z_1, z_2, z_3, t)$

Use the command

```
> dd:=[1,1,2]:MULTIVAR_COVARIANTS(dd);
```

$$\frac{{z_2}^2{z_1}^2{z_3}^2t^2 + tz_3{z_2}^2z_1 - tz_3z_2 - z_2z_1z_3 + z_2z_1t^2z_3 + tz_3{z_1}^2z_2 - z_1tz_3 - 1}{(-1+z_3t^2)(-1+{z_3}^2)(-1+tz_2)(-1+z_3{z_2}^2)(-1+z_1t)(-1+{z_1}^2z_3)(-1+z_2z_1)}$$

Compute $\mathcal{P}(\mathcal{I}_{(4,4)}, z_1, z_2, t)$

Use the command

```
> dd:=[4,4]:MULTIVAR_INVARIANTS(dd);
```

$$-\frac{{z_1}^4{z_2}^4 + {z_2}^2{z_1}^2 + 1}{(-1+{z_2}^2)(-1+{z_2}^3)(-1+{z_1}^2z_2)(-1+z_1z_2)(-1+z_1{z_2}^2)(-1+{z_1}^2)(-1+{z_1}^3)}$$

References

[1] Бедратюк Л.П. Ряди Пуанкаре мультиградуйованих алгебр SL2-інваріантів // Український математичний журнал. — 2011.— т.63. — №6. — с.755 – 763

[2] L. Bedratyuk, The bivariate Poincaré series for the algebra of covariants of a binary form// ISRN Algebra. — 2011. — Article ID 312789. – doi: 10.5402/2011/312789.

[3] L. Bedratyuk, Multivariate Poincaré series for algebras of SL_2 -invariants //C. R. Acad. Bulg. Sci. — 2011. — Vol. 64. — No 6. — p.807 – 814.

[4] L. Bedratyuk, The Poincaré series of the algebras of simultaneous invariants and covariants of two binary forms // Linear and Multilinear Algebra. — 2010. — Vol. 58(6). — P. 789 – 803.

[5] L. Bedratyuk, Weitzenböck derivations and the classical invariant theory, I: The Poincare series // Serdica Math. J. — vol. 36. — No 2. — 2010. — P.99 – 120.

[6] Ilash N. The Poincaré series for the algebras of joint invariants and covariants of n linear forms / C. R. Acad. Bulg. Sci. — 2015. — V. 68. — P. 715 – 724.

[7] T.A. Springer. On the invariant theory of SU(2) // Indag. Math., — 1980. — 42. — P. 339 – 345.

ПАКЕТ MAPLE ДЛЯ ОБЧИСЛЕННЯ РЯДІВ ПУАНКАРЕ

Бедратюк Л., Бедратюк А.

РЕЗЮМЕ

Ми пропонуємо пакет Maple`Poincare_Series` для розрахунку рядів Пуанкаре для алгебри інваріантів/коваріантів бінарних форм, для алгебри спільних інваріантів/коваріатів декількох бінарних форм, для ядра Weitzenböck похідних, для двоваріантного ряду Пуанкаре з алгебри коваріантів бінарної d -форми та для багатоваріантного ряду Пуанкара з алгебри спільних інваріантів/коваріатів декількох бінарних форм.

Ключові слова: інваріант, коваріант, похідна, бінарна форма, ряд Пуанкаре, алгебра інваріантів.

ПАКЕТ MAPLE ДЛЯ РАСЧЕТА РЯДОВ ПУАНКАРЕ

Бедратюк Л., Бедратюк А.

РЕЗЮМЕ

Мы предлагаем пакет Maple `Poincare_Series` для расчета рядов Пуанкаре для алгебры инвариантов / ковариантов бинарных форм, для алгебры общих инвариантов / ковариатов нескольких бинарных форм, для ядра Weitzenböck производных, для двувариантного ряда Пуанкаре по алгебре ковариантов бинарной

d -формы и для многовариантного ряда Пуанкаре по алгебре общих инвариантов / ковариатов нескольких бинарных форм.

Ключевые слова: инвариант, ковариант, производная, бинарная форма, ряд Пуанкаре, алгебра инвариантов.